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ABSTRACT

This paper reviews the literature concerning the development of deductive reasoning, emphasizing the verbal form of the arguments involved in the reasoning. Empirical research is grouped under the heading: assessment studies, critical thinking, instructional studies, and logic and proof. A conceptual framework is developed and a section on errors in deductive reasoning is presented. Also included is a discussion of the "state of the art" and some suggestions for future research. (Author/LS)

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THE DEVELOPMENT OF DEDUCTIVE REASONING:

A REVIEW OF THE LITERATURE

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THE DEVELOPMENT OF DEDUCTIVE REASONING

In their writings on the development of logical thinking ability, Inhelder and Piaget (1958) maintain that class reasoning is a characteristic of the concrete operational period (about ages 7-11), whereas conditional reasoning is not possible before the formal operational period (about age 12 onward). This suggests that the ability to handle deductive arguments is not generally available before adolescence.

All of the studies to be reported in the paper that follows bear in some way on the above concern. Inhelder and Piaget describe this logical development globally, while much of the research has been concerned with the possibility of differential development for specific principles and content areas within the two basic types of reasoning. Tables 1 and 2 illustrate these varieties of reasoning. We are interested here primarily in the verbal form of the arguments. The empirical research will be reported under four main headings: Assessment Studies; Critical Thinking; Instructional Studies; and Logic and Proof. This will be followed by a short review of miscellaneous studies and a section on error in deductive reasoning. Research reported prior to 1955 will receive only cursory attention.

Introduction

The work of Inhelder and Piaget (1958) referred to above is widely known and is also available in such secondary works as Flavell (1963). The primary characteristics of Piaget's period of formal operations which

are not among the characteristics of the earlier period of concrete operations are (1) reversal of the realm of the real with the realm of the possible; (2) hypothetico-deductive reasoning; (3) propositional thinking; and (4) combinatorial reasoning. Adolescents view the realm of the real as being a subset of the realm of the possible, whereas children in the concrete operational period view the possible as an extension of the real. This changed orientation implies a hypothetico-deductive cognitive strategy which tries to determine reality within the context of possibility. Furthermore, the adolescent manipulates propositions or assertions rather than the raw data themselves. He is able to systematically and exhaustively combine the propositions relevant to a problem (Flavell, 1963, pp. 204-211).

To further explicate the notion of combinatorial reasoning, Piaget suggests that adolescents are capable of conceiving possible associations of two elements. These are his sixteen logical operators:

1. $(p \cdot q) \vee (p \cdot \bar{q}) \vee (\bar{p} \cdot q) \vee (\bar{p} \cdot \bar{q})$
2. $p \cdot q$
3. $p \cdot \bar{q}$
4. $\bar{p} \cdot q$
5. $\bar{p} \cdot \bar{q}$
6. $(p \cdot q) \vee (p \cdot \bar{q})$
7. $(p \cdot q) \vee (\bar{p} \cdot q)$
8. $(p \cdot q) \vee (\bar{p} \cdot \bar{q})$
9. $(p \cdot \bar{q}) \vee (\bar{p} \cdot q)$
10. $(p \cdot \bar{q}) \vee (\bar{p} \cdot \bar{q})$
11. $(\bar{p} \cdot q) \vee (\bar{p} \cdot \bar{q})$
12. $(p \cdot q) \vee (p \cdot \bar{q}) \vee (\bar{p} \cdot q)$
13. $(p \cdot q) \vee (p \cdot \bar{q}) \vee (\bar{p} \cdot \bar{q})$
14. $(p \cdot q) \vee (\bar{p} \cdot q) \vee (\bar{p} \cdot \bar{q})$
15. $(p \cdot \bar{q}) \vee (\bar{p} \cdot q) \vee (\bar{p} \cdot \bar{q})$
16. $(p \cdot q) \vee (p \cdot \bar{q}) \vee (\bar{p} \cdot q) \vee (\bar{p} \cdot \bar{q})$

The ability to generate these combinations is at least a necessary condition for formal thought. The discussion of the research literature will begin by reviewing studies which have attempted to assess formal reasoning ability at various age levels. The concern here is narrowed to deductive reasoning and the term 'logic' is not to be confused with the use of that term in the broader Piagetian context, e.g., as in Logical Thinking in Children, the well known collection of studies by Sigel and Hooper (1968) or Piaget's Logic and Psychology, (1957).

Assessment Studies

Piaget's hypotheses suggest that conditional, i.e. if-then reasoning, does not appear until the period of formal operations. In an effort to examine Piaget's notions more closely, Hill (1961) explicitly sought developmental patterns for specific valid principles of class, conditional, and syllogistic reasoning. She found, contrary to Piaget, that children at ages 6,7, and 8 were able to recognize valid conclusions deduced from given sets of premises. She reported steady increases in this ability with age for all three types of logic examined. Her data indicated that conditional logic was easier than class logic at age 6, a difference which disappeared by age 8.

Hill's findings have been widely quoted, perhaps because she appeared to contradict Piaget. Furthermore, this apparent contradiction engendered much research in an effort to extend Hill's findings. The early work of O'Brien and Shapiro (1968, 1970) stems directly from this concern. These researchers evaluated the ability of children ages 6 to 13, with no

explicit instruction in logical reasoning, to test the logical necessity of a conclusion when no logically necessary conclusion existed. They found, as Hill suggested, that these children had considerable success in recognizing logically necessary conclusions and that this ability leveled off high in the 6-to 8-year-old range. However, these same children experienced great difficulty in testing the logical necessity of a conclusion, i.e., in recognizing invalid conclusions. Further, they exhibited slow growth in this ability from ages 6 to 13 and showed no evidence of any periods of leveling off over this eight-year span.

In a subsequent study, O'Brien, Shapiro, and Reali (1971) concluded that an understanding of implications expressed in "if-then" language should not be taken for granted in students at Grade 10 (about age 16) or below. Children in this study consistently followed what the researches referred to as 'Child's Logic'. A child is said to be using "Child's Logic" when he interprets "if-then" statements as "if and only if" statements, in contrast to 'Math logic'. In Piaget's symbolism, the subject seems to construct $(p \cdot q) \vee (\bar{p} \cdot \bar{q})$ for $p \rightarrow q$, rather than the correct $(p \cdot q) \vee (\bar{p} \cdot \bar{q}) \vee (\bar{p}\ q)$, (p. 203). Two major questions of this research (and later researchers by O'Brien) were (1) "Is 'Child's Logic' attributable to subjects' inability to construct the threefold combination for $p \rightarrow q$...?" and (2) Do subjects regard merely the form of an argument or do they attend differently to items according to the content they contain (pp. 203, 205)? The first question was examined by using a test which omitted the "if-then" language in favor of a logically equivalent form. They concluded that "...the ability to construct and keep in mind the complex combination

$(p \cdot q) \vee (\bar{p} \cdot \bar{q}) \vee (p \cdot q)$ given $p \rightarrow q$ ... is more widely available than it is used in "if-then" situations. 'Child's Logic' is not due wholly to an underlying cognitive inability in subjects. The use of "if-then" language suppresses students' application of their combinatorial ability (p. 216)."

This particular study (O'Brien, et.al., 1971) examined only conditional reasoning and presented no breakdown of results by principle of inference (converse and inverse were used). The variable context, however, was an important one. Items labeled causal and class inclusion were used.* The researchers concluded that their results indicate that "many subjects do not act on the form of items without regard for the context. In class inclusion items the existence of $(\bar{p} \cdot q)$ under $p \rightarrow q$ is much more readily acknowledged than in items which are causal in context (p. 216)."

The question of context was further examined by O'Brien (1972) in his study of logical thinking in adolescents in which he analyzed the interactions of context, grade level, mode, and form. By 'form' is meant the principles of inference, in this case Modus ponens, contraposition, inversion and conversion where items of the latter two types are referred to as "open" since a judgment on their validity requires a "not enough clues" or "maybe" response. 'Mode' refers to the position of negatives in the implication: $p \rightarrow q$, $\bar{p} \rightarrow q$, $p \rightarrow \bar{q}$, or $\bar{p} \rightarrow \bar{q}$. Again in this study there was a consistent imbalance between subjects' scores on closed and open items, but it appeared as though the use of 'Child's Logic' does not explain all the data. In particular, it does not "explain subjects' wide-

* Causal items suggest a cause-effect relationship between antecedent and consequent: "If Jack will play, the Cougars will win." Class exclusion items suggest a class inclusion relationship: "If the house is big, then it is white."

spread inability to recognize the validity of contrapositive inference nor does it explain any inverse-converse imbalance that might exist. We suspect, then, the existence of a different 'Child's Logic', a semi-'Child's Logic', in which p→q is interpreted as 'p yields q and q yields p and nothing else' (p.427)." Thus, in this study there was a difficulty with the contrapositive and an inverse-converse imbalance not noted in the earlier studies (O'Brien, 1968, 1971).

Yet another study by O'Brien (1973) focused attention on performance on the four inference patterns, rather than simply open and closed items, by college students. It was reasonably felt that these subjects would have attained the formal operational period. Items were causal or class inclusion in context. O'Brien found widespread use of 'Child's Logic' among these college subjects. Consistent use of 'Math Logic' was employed by very few students even in the face of a college-level course in logic.

Research dealing with some of the same variables as that by O'Brien and his collaborators has been carried out by a number of investigators, who, because of their background, will be discussed in the next section. Still other singleton studies have been reported. Particularly noteworthy is the research conducted by Donaldson (1963) in England. Donaldson examined children age 11 to 14 and found that their ability to infer valid conclusions in class logic increased with age, but that their ability to recognize invalid inference patterns showed no improvement over the same period. Furthermore, she suggests that children mix common sense causal reasoning (true premises and conclusion) and formal reasoning. If

a child reasons causally, this does not imply that he cannot reason formally (Donaldson, 1963, p. 204). In other words, an understanding of all possible combinations may not be required for the reasoning to be formal in a simple situation. Donaldson's interesting discussion of errors is taken up elsewhere in this paper.

Similarly, Howell (1966) reported that accelerated junior high school students were able to recognize some valid principles of conditional reasoning, but could not recognize invalid principles. Miller (1955) administered a 'fallacy recognition test', to students in Grades 10, 11, and 12 and found that there existed a rank order of difficulties for the fallacies appearing in the test. Moreover, the ability to recognize fallacies was not significantly affected by grade level, sex, scholastic standing, mental age, or reading level of the students tested (Miller, 1955, p. 127). Unfortunately, no clear definition was given of what it means to recognize a fallacy.

There is strong evidence in the studies discussed above of the difference between the ability to recognize and the ability to test logically necessary conclusions. Thus, despite the widespread use of Hill's findings to counter Piaget, caution is called for in the interpretation and application of her findings.

The Critical Thinking Framework

The seminal work of Ennis (particularly 1959, 1962) in the area of critical thinking has led to the development of a conceptual framework within which a large number of investigations dealing with deductive

reasoning have been carried out. The original work aimed at a concept of critical thinking based on a root notion of the 'correct assessing of statements' (Ennis, 1962, p.83). Twelve aspects of critical thinking were delineated, three of them bearing directly on the concerns of this review, namely, (1) judging whether statements contradict each other; (2) judging whether a conclusion follows necessarily; and (3) judging whether something is an assumption. It would appear, then, that the ability to judge a deductive argument is a necessary (but not sufficient) competence for critical thinking. From this framework have stemmed a number of studies directly or indirectly under the aegis of Ennis and his Cornell Critical Thinking Project (Ennis is now at University of Illinois and it has become the Critical Thinking Project). Still others have followed through his students and their students in turn. All, however, share this common framework and are, therefore, considered together.

One of the first of these investigations to examine deductive reasoning is the extensive study by Ennis and Paulus (1965) into critical thinking readiness. After a review of the literature, including that of Piaget, showed that little research attention had been given to the differential development of specific deductive principles, these researchers focused on an assessment of this development in both class and conditional types of reasoning. They found large differences in the percentages of mastery for specific principles of class and conditional reasoning at given grade levels from grades 4 through 12. Their data indicated that the greatest improvement with age in the mastery of specific principles was that for the invalid principles, inversion and conversion. It appeared that class

logic was easier than conditional logic across all grade levels. In addition, Ennis and Paulus concluded that although younger children did not benefit from instruction in class logic, it could be taught successfully from age 11 or 12 onward. Considerable mastery of class logic was shown by 17- and 18-year-olds without teaching. Even with conditional logic explicitly taught, not much improvement was evident until age 16 - 17, when great strides were made particularly in the ability to judge the fallacy principles. It seems that grade 11 subjects were ready to master these principles whereas pupils in grades 6 and 7 were not ready.

While this study (Ennis and Paulus, 1965) was not undertaken as a direct test of Piaget's claims about the development of logical thinking, the work of these investigators suggests several important questions:

(1) Is there actually a development of logical ability as children grow older?

(2) Does this development (if there is any) come in stages?

In summary of the above results, the authors found definite evidence that logical ability develops in time as children mature. Furthermore, they found the aforementioned developmental patterns in class and conditional logic. At ages 10 - 12, the principles expressing basic fallacies seemed to be most difficult. A major area of differences found in the developmental patterns was concerned with the concrete familiar, symbolic, and suggestive* components of the tests. The three components of the conditional reasoning test were of about equal difficulty at each grade level. On the

* A number of researchers consider three components of a 'content-dimension' variable: (1) Concrete - familiar items are those in which the conclusion of the argument possesses a neutral truth value, while the vocabulary of the argument in both premises and conclusion is familiar; (2) Suggestive items contain at least one statement which contradicts common knowledge; (3) Abstract (or symbolic) items contain non-verbal symbolism.

other hand, the concrete familiar components on the class reasoning test seemed to be easier than each of the other components, especially the symbolic component.

In answer to question #2, the researchers felt that what one counts as a "stage" is crucial. However, they reported that gross plateaus extending over a period of years were not evident. The report of this study includes a chapter on Piaget's Logic in which a conceptual analysis and comparison with standard logic is begun (see also Ennis, 1969; Parson, 1960). The authors conclude that although they find ambiguities and inconsistencies in Piaget's writings on logic, his conceptions, (as far as he goes) are near enough to their own to be grossly equated.

As an assistant working with the Cornell Critical Thinking Project, Gardiner (1965) investigated the understanding of the meaning of logical operators in propositional reasoning. He suggests that the rules of logic "are essentially the meaning of certain groups of words within a language called logical operators. Those logical operators state a relation between two propositions, eliminate a subset of alternatives within the propositional structure and determine the truth-status (true of false or undetermined) of one proposition on the assertion or denial of the other (pp.4-5)."

He thus returns to the propositional structure of Piaget, i.e. the premises of any argument "may invariably be interpreted as the elimination of one or more alternatives from this exhaustive list (p.7):"

$$p \cdot q \vee p \cdot \bar{q} \vee \bar{p} \cdot q \vee \bar{p} \cdot \bar{q}$$

Gardiner constructs a set of twelve rules (p. 9a) dealing with logical operators and defined in terms of Piaget's sixteen combinations. He suggests

that the order of difficulty of the rules over subjects "may provide some insight into the order of acquisition within subjects. If the order of acquisition of the rules is consistent from subject to subject, by placing the rules in order of difficulty, it is possible to predict from the number of rules understood by a subject, the particular rules understood by that subject. The percentage of such predictions which would be correct may be calculated by means of a Guttman analysis. For the 277 subjects and 14 rules,* this percentage was found to be .87. This index is not meaningfully interpretable in terms of probability by chance but it does suggest a certain consistency from subject to subject in the development of understanding of logical rules (p.17)." This concern with the order of acquisition of the logical operators by subject has also been investigated by Airasian, et. al. (1973), and is discussed elsewhere in this paper.

Paulus' (1967) work supports the findings of other researchers with respect to the differential development among various principles of conditional reasoning. He reported that the invalid principles were mastered by less than 20% of the students at all but grades 10 and above. He also found empirical support for the hypothesis that deducing is more difficult than assessing the conclusions of deductive argument; and that deducing is learned later chronologically than assessing (p. 131). There were indications, however, that deducing is not consistently more difficult than assessing over all principles and types of content. Roberge (1970) studied 4th through 10th graders' abilities to reason with basic principles of class and conditional reasoning. He found that neither type of

* See Gardiner (1965, p.13). Two additional rules were constructed by combinations of other rules.

reasoning was consistently easier at all grade levels, although the combined mean for class reasoning was significantly larger than that for conditional reasoning. In a related study Roberge and Paulus (1971) focused on the differential effects of content (concrete-familiar, suggestive, and abstract) on development of mastery. In general, they found that the difficulty of applying a principle of reasoning increases as the meaningfulness of the content decreases. The order of difficulty, from least to most difficult, seems to be (a) concrete-familiar; (b) suggestive (misleading), and (c) abstract. The earlier study by Paulus (1967), however, does provide an exception to this hierarchy. Paulus found that subjects in Grades 5, 7, 9, and 11 had as much difficulty assessing conclusions of logical arguments containing familiar terms as with those containing abstract terms, and that these students found it easier to deduce conclusions from abstract rather than familiar premises. The investigation by Tripp (1973) comments further on this important issue.

The question of order of difficulty has appeared in a number of studies. Jansson's (1973) study of pre-service elementary school teachers is supportive of the findings of Roberge (1970) and O'Brien (1972) that the easiest principle of reasoning is modus ponens, followed by contraposition (modus tollens), over grade levels. These studies agree that the two invalid principles, inversion and conversion, are more difficult, but which of the two is easier has not been clearly determined, as even a cursory glance at Tables 3 and 4 will show. The Jansson study indicates that the differences in ability due to type and principle have not disappeared even in college level subjects. Roberge (1969) found that negation in the major premise of an argument has "a marked influence on the

development of logical ability in children (p. 721)." This effect appeared to be consistent over grade levels (grades 4,6,8, and 10) for concrete-familiar items, but varied according to the type of reasoning. The means for class reasoning in this study were significantly smaller than those for conditional reasoning where a negation actually appeared in the major premise. The investigator suggests that this may be due to the essentially ambiguous construction "'all...not' as in All that glitters is not gold (p. 721)." This is clearly an area for further study and has been touched on by at least two recent studies (O'Brien, 1972; Jansson, 1974).

The series of investigations reviewed in this section has focused principally on the following variables: sex, age (grade level), type of reasoning, (class and conditional), principle of reasoning (see Tables 1 and 2), and kind of content as viewed with the Cornell Critical Thinking framework. Tables 3 and 4 provide a crude summation of the results with respect to type and principle. The findings with regard to content are somewhat ambiguous, while a universally consistent finding is that sex is a negligible factor in the development of logical reasoning ability.

Instructional Studies

The studies concerned with explicit instruction in logical reasoning fall into a number of categories seemingly unrelated to each other except that they examine logical reasoning. The first such set of studies is that which stems from Suppes and his work at Stanford. These will be dealt with very briefly because their concern is with symbolic logic, not the primary focus of the present review.

The study by Suppes and Binford (1965) has been widely quoted in the literature on the basis of an article appearing in the Arithmetic Teacher. The objective of the project reported there was to investigate the difficulty and suitability of teaching elements of logic to fifth and sixth grade students. Material in mathematical logic was prepared and taught to academically talented fifth and sixth grade students by teachers specifically trained to deal with instruction in formal symbolic logic in the elementary school. In the experimental investigation two Stanford University Logic classes served as control groups. The study concluded that upper quartile [of the group tested or of the total population?] elementary school pupils can achieve a significant and technical mastery of elementary mathematical logic. The level of mastery is 85-90% of that achieved by the University students. Furthermore, the amount of study time for the school pupils was not significantly greater than that needed by the college students, but was spread out over a longer time period. Reported anecdotal evidence from teachers suggests that there is some carry-over into other fields, especially arithmetic, reading, and English. The details given for this project are minimal and the sample not representative. Any extrapolation to other situations would appear to this writer to be unwarranted.

A number of other studies reported in Technical Reports of the Stanford Institute for Mathematical Studies in the Social Sciences deal with the teaching and understanding of formal symbolic logic. The reports by Goldberg (1971) and Goldberg and Suppes (1972) describe respectively a computer-assisted instructional system for elementary

mathematical logic and a program on this system for exercises in finding axioms. This system was used by Kane (1972) in his attempt to define criteria for classes of equivalent proofs, by examining variability in actual college student proof behavior. A technical report by Moloney (1972) likewise dealt with college students in a logic course on a CAI system at Stanford. A major objective was to identify some of the structural features of an elementary logic curriculum which affect problem-solving difficulty. Stepwise linear regression was used to develop a model which would account for problem difficulty as a function of problem characteristics.

Of more general interest are those studies of logical thinking which avoid the formal symbolism and deal with class and conditional arguments in verbal form. A major study in this area is the one by Ennis and his collaborators (1969) entitled Conditional Logic and Children, prepared under the auspices of the Cornell Critical Thinking Project. While considered a readiness study by its authors, the experimental part of the investigation involved the construction of elaborate instructional episodes designed to help children in Grades 1, 2, and 3 learn four principles of conditional logic: inversion, conversion, contraposition, and transitivity. The questions that the investigators were attempting to answer dealt with readiness to learn more, and a consideration of the Piagetian hypothesis that logical ability is correlated with age. As indicated earlier in the paper, Ennis and his co-workers are not always sure what Piaget means by many of his claims, but they do share the Piagetian belief that a crucial factor of deductive ability is the ability

to assume and reason from that which might not be believed.

There is not space here to describe the instructional materials and environment used in this study. Suffice it to say that conditional reasoning was the focus and final testing consisted of several pairs of conditional statements, the first of which was suppositional in nature, i.e., the student had to pretend that something was true and the second question was factual in that the student could see that some premise was true. In a control group great variation in the ability to handle conditional logic was evident. While possessing some ability with suppositional premises, these subjects were better at dealing with factual ones. Only a weak relationship between ability in conditional logic and age was found. The children in the experimental group appeared not to benefit from the use of the teaching materials. Of course this does not imply that children of this age are incapable of learning conditional logic. As the authors suggest, it may merely mean that the instructional materials used were not adequate.

Two further studies which stem by a very circuitous route from the Ennis conceptual framework are those by Tripp (1973) and Shipman (1973) dealing with instruction of pre-service elementary school teachers in a computer assisted instruction (CAI) setting. In both of these studies we are dealing with subjects clearly in the formal operational period, at least according to chronological age. That they are actually formal operational is, of course, open to question (O'Brien,1973; Jansson, 1973).

The Shipman investigation was an attempt to specify, develop, and validate an instructional hierarchy useful to mathematics educators in assisting prospective teachers to learn to judge the validity of simple

verbal arguments of the conditional type. The hierarchy consisted of two major independent sub-skills: (1) the ability to correctly translate the argument from verbal to symbolic form, and (2) the ability to correctly judge the validity of an argument given in symbolic form (using a Venn diagram technique). The study compared instructional sequences (1) - (2) and (2) - (1) both with and without a Gagné-type Guided Thinking Information episode. Most subjects learned to translate from verbal to symbolic for all the principles of inference used in the study, namely, conversion, contraposition, and transitivity. Furthermore, the sequence of learning to judge the argument given symbolically followed by the translation skill appeared more effective than the reverse instructional sequence for attainment of the terminal objective. In this and the following study (Tripp, 1973) adequacy criteria were established beforehand for all instructional episodes. The instruction on the translation skill was deemed adequate while the episode involving judgment was of questionable adequacy. Thus some of the findings are tenuous and due to a post hoc analysis which ignored the adequacy criteria, since the design permitted no hypothesis testing in the absence of adequate instruction.

Tripp (1973) investigated the transfer of the skills learned in the instructional material just described to (1) the principleof inversion (concrete-familiar), and to (2) the principles of conversion, contraposition and transitivity in the suggestive content domain. Tripp's findings suggest tentatively that, at least in the concrete-familiar content domain, it may be sufficient to instruct on the three principles, contraposition, conversion, and transitivity in order to attain mastery of these three <u>and</u> inversion. A further finding of this study, which

contradicts the earlier findings of Roberge and Paulus (1971) with respect to school age children, was that the transitivity principle was significantly easier in the suggestive content domain and that this was true on both pre- and post-tests, i.e., it was either independent of or reinforced by explicit instruction.

Weeks and McKillip (1971) have reported a study in which they used attribute block training for 90 minutes per week, for eight weeks, with second and third graders. Their instructional program followed guidelines proposed by Z.P. Dienes, while the remainder of the subjects' mathematics was the standard fare. The results of their analyses "indicated that the attribute block training had a strong positive effect at both grade levels... in the development of logical reasoning ability... The effect in Logical reasoning ability due to attribute block training was more apparent in the development of quantificational logic than in the development of sentential logic."

This type of finding, however, is not confirmed by all investigations. While a number of reporters (e.g. Hill, 1961; Roberge, 1972) have suggested the introduction of brief units on class and conditional logic into the mathematics curriculum as early as Grade 4, Carroll (1970), does not echo this recommendation without qualification. She introduced instruction in the understanding of the four basic forms of logical inference to Grade 9 low achievers in mathematics. Her results indicated no significant improvement in the combined score for all forms.

Logic and Proof

Mathematics educators, perhaps more than others, have taken particular interest in the development of deductive reasoning per se. This is undoubt-

edly due to the role that such reasoning plays in their discipline, particularly in the construction of mathematical proofs. Morgan (1970) reported an investigation of college mathematics students' abilities in proof-related logic. He found that mathematical experience was not a sufficient condition (for his sample) for learning all the patterns of conditional reasoning investigated, namely, recognition of (1) equivalence of a conditional statement and its contrapositive; (2) invalidity of the inverse and converse of a conditional statement, and (3) the starting assumption for a direct proof, contrapositive proof, and a proof by contradiction. He concludes that further research is needed to determine the role of ability in logic and proof in the successful completion of an undergraduate program in mathematics. Moreover, since this sample included prospective teachers, the adequacy of their preparation is again questioned.

Lovell (1971) reported on an investigation by Reynolds into "the development of the understanding of mathematical proof in British selective (grammar and technical) secondary schools, to see how well this development is explained by the framework provided by Piaget's genetic psychology (p.66)." Reynolds viewed proof strategy as consisting of two parts: construction of a hypothesis and construction of a proof. Students tested were in Forms 1,3,5, and 6, i.e. 12-13, 13-14, 15, and 17 years old respectively. Of interest for this review are the results dealing with deductive reasoning as such, although other aspects of proof, including generalization, symbols and assumptions were included in the original study. Subjects in the early forms had difficulty with

the invalidity of the converse, but by the sixth form, 98% had mastered this. Similarly, the ability to deduce a conclusion from a given set of premises (verbal, non-mathematical) increased from 32% in form 1 to almost 80% in form 6. Reynolds, in his anecdotal reporting, also cited examples where the construction and testing of hypotheses did not develop together, although presumably this is not unexpected.

Other Studies

A number of studies not explicitly cited in this paper are listed in the "Additional References" at the end. Two studies should be mentioned, however, which nevertheless do not fit into the above loosely defined categories. Jansson (1974) has reported on an initial exploratory study which attempted to account for the difficulty in judging simple verbal deductive arguments in terms of problem characteristics, both structural and linguistic. While using a linear regression model similar to that developed in some of the Stanford Studies (c.f., Moloney, 1972) and the problem-solving work of Jerman and Rees (1972), this investigation operated in a non-CAI, non-symbolic logic setting. While the viability of this research as a long-term undertaking is still uncertain, further exploratory work is anticipated.

A study by Airasian, Bart, and Greaney (1973) attempted to "examine [Piagetian] intra-period cognitive development by determining the extent to which tasks requiring skills proper to a single period, formal operations, were hierarchically or sequentially ordered (p. 1)." That is, the study sought to determine if the sixteen logical propositions of Piaget were hierarchically ordered in the sense of mastery of one being a precondition

for mastery of another. Ordering theory, an extension of scalogram analysis which can be non-linear, was used to determine prerequisite relations between pairs of logical propositions. Using 14-year-old subjects, the investigators present their empirically derived 'ordering among logical statements'. They state two major conclusions from this research. First, "disjunctive propositions such as implication and disjunction which are unions of three atomic propositions are indicated to be more difficult to understand than conjunctive propositions such as non-implication and conjunction. This finding is compatible with similar results on the psychological order of difficulty for propositions established by Bruner [Goodnow and Austin] (1956) and Bart (1969, p.7)."

Further, the writers report that "the understanding of atomic propositions such as conjunction and non-implication are in general indicated to be precondition to the understanding of bi-atomic propositions such as equivalence and affirmation of p which are compositions of two atomic propositions. Also, the understanding of bi-atomic propositons is in general indicated as a precondition to the understanding of tri-atomic propositions such as implication and compatibility. This finding lends credence to the hypothesis that the psychological order of comprehension for propositions has an additive atomic basis (pp. 7-8)."

Airasian's work is not unlike that of Gardiner (1965) cited earlier, but appears more structurally oriented and independent of the conceptualizations of the Critical Thinking Project approach. His findings would seem to call out for replication. The ordering theoretic approach with its allowance for a non-linear ordering seems preferable to the straight

scalogram analysis, but as Kofsky (1968) points out, any type of rank ordering for a given set of items may not correspond to the sequence in which they were mastered (or should be taught).

Error in Deductive Reasoning

This review would be incomplete without some further reference to the cause of error in deductive reasoning. The issue is at once philosophical, psychological, and psycho-linguistic and deserves a paper by itself. Perhaps the most frequently cited paper is that by Henle (1962), "On the Relation between Logic and Thinking." Henle begins:

> The question of whether logic is descriptive of the thinking process, or whether its relation to thinking is normative only, seems to be easily answered. Our reasoning does not, for example, ordinarily follow the syllogistic form; and we do fall into contradictions. On the other hand, logic unquestionably provides criteria by which the validity of reasoning may be evaluated. Logical forms thus do not describe actual thinking, but are concerned with the ideal, with 'how we ought to think'. And yet a problem seems to be concealed beneath this easy solution (p. 366).

Older writers such as Kant and Boole regarded "Logic as the science of the laws of thought. (Cohen & Nagel, 1934)." Henle sums up her view of the position of contemporary investigators, particularly psychologists, by reference to the statement by Bruner, Goodnow, and Austin (1956) that people seem to prefer "empirically reasonable propositions" rather than logical ones. In other words, logical principles are assumed "irrelevant, if not antithetical, to actual reasoning." In her own position Henle sides more with the older view and suggests four processes that lead to error in deductive reasoning:

1) Failure to accept the logical task. Error results from the subject's judging the factual content of the conclusion rather than the logical form of the argument.

2) Restatement of a premise or conclusion so as to change the meaning. Henle suggests that we should judge a subject's reasoning on the argument the subject actually used.

3) Omission of a given premise.

4) Slipping in of additional premises.

Thus, errors in judging the validity of deductive arguments need not involve faulty reasoning. One is, of course, tempted to ask why subjects apply (albeit unknowingly) any one of the four processes which lead to error. Is it something in the material which focuses attention elsewhere or is it an innate inability to handle deductions qua deductions? The 'why' questions, of course, continue.

Donaldson, in the study referred to earlier (1963), suggests several reasons for error (pp. 204-212), not unlike those suggested by Henle. In particular, those errors classified as arbitrary are quite similar, namely, (a) the subject ignores the available information (for some reason), and (b) the subject adds information to that given. A second category of error is referred to by Donaldson as executive. This is error caused primarily by an overloading of the nervous communication channels.

Such explanations as those given by Donaldson and Henle beg the question of the role of cognitive development, particularly when we are dealing with non-adults. The studies by O'Brien and his collaborators (1971, 1972, 1973) cited earlier have considered this question. Their findings suggest, for example, that language is an important variable,

and that subjects' tendency to use 'Child's Logic' were due partially to language and partially to cognitive inability. From a Piagetian point of view explanations such as that of Henle or Bruner, Goodnow, and Austin, would appear somewhat simplisiic, or at least notapplicable to the non-adult.

Studies of several decades ago (e.g., Woodworth and Sells, 1935; Sells, 1936) explained reasoning error by college students in terms of the 'atmosphere effect'. Briefly, the atmosphere effect in syllogistic reasoning may be described (Chapman and Chapman, 1959) as the "drawing of conclusions on the basis of global impressions of the premises. Thus an affirmative premise, i.e., 'all are' or 'some are' produces an affirmative atmosphere and a negative premise, i.e., 'none are' or 'some are not' produces a negative atmosphere (p. 84)." These investigators in their report and discussion suggest that their data refute Sells' conclusions, and further, that his findings were an artifact of his test format. Explanations given by the writers include: 1) the possibility of reasoning by probable inference, and 2) the fact that many subjects assume the converse of a statement to be true (see also O'Brien).

Wason (1964) likewise accepts the inductive inference explanation of errors made in syllogistic reasoning. He investigated the effect of self-contradiction on fallacious reasoning. When faced with a valid inference contradicting a previously made invalid inference, subjects tended to withhold further invalid inferences. "This suggests that inconsistency allowed the subjects to gain insight into the fallaciousness of their reasoning... In addition, it is clear that the effects of

inconsistency tend either to work when it is first introduced, or not at all (p. 129)." Wason's subjects were university students and thus presumed to be in the formal operational period.

Concluding Remarks

The paucity of sound research into instruction in the area of deductive reasoning is evident. The number of studies is extremely small. Hopefully, as more data are accumulated relative to readiness and general assessment, this situation will change. It appears, however, witness the Ennis (1969) study, that investigation in the two areas, assessment and instruction, must move forward hand in hand if real progress is to be made in our understanding of formal reasoning.

Before expanding on the more-research-needs-to-be-done theme, let us attempt to summarize the data already at hand in the studies reviewed here. Tables 3 and 4 present percentage mastery data for those studies which report such information. The reports by O'Brien and coworkers give only percent of items correct and not percent of students who master a given principle of inference. "Mastery of a principle" is operationally defined in each study as a proportion of items of the judgment type. It should be noted that the bulk of Table 3 is taken from Roberge (1972); the O'Brien figures in the table are based on the writer's calculations, using a weighted mean of O'Brien's (1973) Causal and Class inclusion categories. Roberge (1972) provides a more detailed discussion of the data in his table.

Naturally the figures in these tables are not strictly comparable. As Beilin (1971) points out in his discussion of the work of Ennis (1969)

and Piaget, the interpretation of results will always be difficult in the presence of different conceptualizations. The fact that a number of the studies presented here relate to the Ennis Critical Thinking conceptual framework provides some basis of comparison among those studies. As Ennis (1965) himself points out, however, comparison with Piaget is difficult at best because of the way in which his theory is explicated. Furthermore, as several studies show, variables other than just type of reasoning and principle of inference are important.

Despite the problems of comparability, certain trends are evident in the existing data. With isolated exceptions, there is improvement with age in the ability to handle these arguments in both class and conditional form. In general, class reasoning is easier at all ages for all principles, with mastery of the fallacy principles appearing later, particularly in conditional form. None of the researchers here noted evidence of plateaus in their data to support Piaget's notion of a leveling out at certain ages, i.e., a 'stage' development. It does seem clear from the data, however, that there is differential development according to principle of inference as well as type of reasoning, although there is some ambiguity concerning the degree of difficulty of the fallacy principles and the order in which they are learned.

The work of Ennis and his collaborators suggests that explicit instruction in the fallacy principles is probably fruitless before mid-adolescence, but that some gain is achieved particularly in class reasoning at an earlier age. It is clear, however, that we do not yet have an accurate profile of how children learn these skills either with or without explicit instruction. A large number of variables play a role,

and it may be that some of these are not yet even identified. Variables which deserve further investigation include content, context, negations, and such linguistic considerations as sentence length, and word length.

Longer, and hence more reliable, tests of each principle would be desirable, even when looking at some of the other variables.

While more data is being collected over a range of ages, further replication of exisitng studies, particularly at the upper and lower age levels is needed. It is evident, however, that great gains have been made in the past decade. In the next decade the present research thrust will hopefully continue, but with a new emphasis on instruction and curriculum building.

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TABLE I

Basic Principles of Class Reasoning

Principle	Validity	Symbolic Form	Concrete Form
1	Yes	All A's are B's x is an A Therefore X is a B	All of the dogs that are brown are named Rover. Therefore the dog's name is Rover.
2	No	All A's are B's x is a B Therefore x is an A	All of the paintings that belong to Don are paintings of horses. This is a painting of a horse This painting belongs to Don.
3	No	All A's are B's x is not an A Therefore x is not a B	All of the cars in the garage that are Mr. Smith's are black. The car in the garage is not Mr. Smith's Therefore the car is not black.
4.	Yes	All A's are B's x is not a B Therefore x is not an A	All of the cats that are not named Tabby are also not white. The cat is white. Therefore the cat's name is Tabby.
5.	Yes	All A's are B's All B's are C's Therefore all A's are C's	All of the coats in the closet that belong to Donna are blue. All of the coats that are blue have white buttons. Therefore all of the coats in the closet that belong to Donna have white buttons.

TABLE 2

Basic Principles of Conditional Reasoning

Principle	Validity	Symbolic Form	Concrete Form
1	Yes	if p, then q p q	If the dog is brown, then his name is Rover. The dog is brown. The dog's name is Rover.
2.	No	if p, then q q p	If the painting belongs to Don, then it is a painting of a horse. This painting belongs to Don.
3.	No	if p, then q not p not q	If the car in the garage is Mr. Smith's, then it is black. The car in the garage is not Mr. Smith's The car is not black.
4.	Yes	if p, then q not q not p	If the cat's name is not Tabby, then she is not white. The cat is white. The cat's name is Tabby.
5.	Yes	if p, then q if q, then r if p, then r	If the coat in the closet belongs to Donna, then it is blue. If it is blue, then it has white buttons. If the coat in the closet belongs to Donna, then it has white buttons.

Table 3

Percentages of Students Who Had Mastered Each of Five Principles of Deductive Reasoning in Conditional Form (Roberge, 1972)

Grade Level	Principles					n/N	p	Researcher(s)
	1	2	3	4	5			
4	53	2	2	35	28	8/12	.02	Roberge (1970)
5	51	2	3	30	25	5/6	.02	Ennis & Paulus (1965)
4-5	59	0	2	16		5/6	.02	Gardiner (1965)
6	54	0	0	23	28	8/12	.02	Roberge
7	56	3	6	41	45	5/6	.02	Ennis & Paulus
6-7	79	0	2	23		5/6	.02	Gardiner
8	95	0	0	74	81	8/12	.02	Roberge
9	66	4	5	35	40	5/6	.02	Ennis & Paulus
8-9	89	4	4	48		5/6	.02	Gardiner
10	100	19	5	65	82	8/12	.02	Roberge
11	62	3	12	35	58	5/6	.02	Ennis & Paulus
10-11	99	30	20	47		5/6	.02	Gardiner
College		40	13	57		8/12	.02	O'Brien (1973)
College	84	11	21	31	78	6/8	.02	Jansson (1973)

Note n = number of items required for mastery of a principle
N = total number of items for a principle
p = probability of correctly answering at least n items by guessing alone.

Table 4

Percentages of Students Who Had Mastered Each of Five Principles of Deductive Reasoning in Class Form

Grade Level	Principle					n/N	p	Researcher(s)
	1	2	3	4	5			
4	56	5	34	10	31	5/6	.02	Ennis & Paulus (1965)
4	60	2	0	44	35	8/12	.02	Roberge (1970)
6	79	15	44	10	37	5/6	.02	Ennis & Paulus
6	58	4	2	44	28	8/12	.02	Roberge
8	91	21	55	15	46	5/6	.02	Ennis & Paulus
8	82	5	4	79	74	8/12	.02	Roberge
10	92	44	80	24	63	5/6	.02	Ennis & Paulus
10	95	40	21	75	79	8/12	.02	Roberge
12	97	68	75	43	67	5/6	.02	Ennis & Paulus
College	91	19	33	27	65	6/8	.02	Jansson (1973)

Note n = number of items required for mastery of principle
N = total number of items for a principle
p = probability of correctly answering at least n items by guessing alone